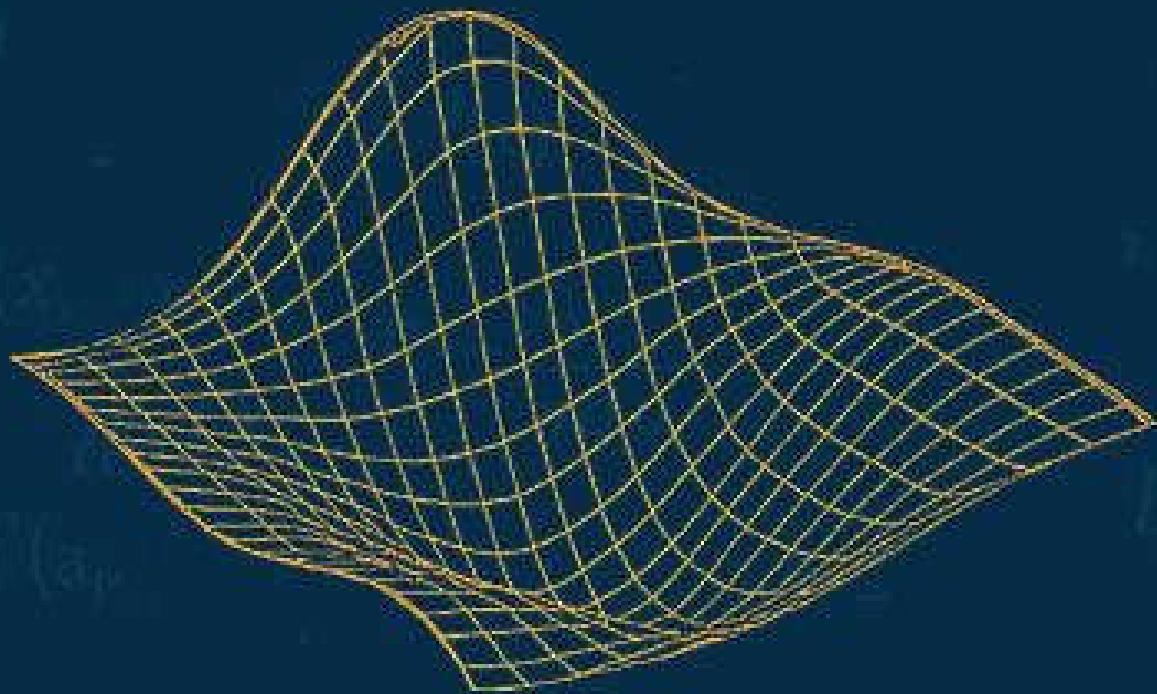


ADVANCED STUDIES

IN MATHEMATICS AND STATISTICS

VOLUME-1



EDITOR

MOBIN AHMAD

ADVANCED STUDIES IN MATHEMATICS AND STATISTICS

**ADVANCED STUDIES IN
MATHEMATICS AND STATISTICS
VOLUME-1**

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Preface

The field of mathematics and statistics continues to evolve, bridging theoretical advancements with practical applications across diverse disciplines. *Advanced Studies in Mathematics and Statistics-I Volume – I* is a comprehensive volume that brings together cutting-edge research and foundational concepts in these fields, offering a rich resource for researchers, graduate students, and professionals seeking to deepen their understanding of advanced mathematical and statistical methodologies.

This book encompasses a broad spectrum of topics, carefully curated to reflect the dynamic interplay between pure and applied mathematics, as well as statistical theory. The chapters cover a range of subjects, from numerical methods for solving non-linear equations to the intricacies of graph theory, ring theory, and fuzzy optimization. Each chapter is designed to provide both theoretical rigor and practical insights, making the content accessible to those with a strong mathematical foundation while also serving as a reference for specialized research.

The book begins with an exploration of numerical solutions to non-linear equations, presenting classical and fast-convergent methods such as Newton's, Steffensen's, and Halley's approaches. These foundational techniques set the stage for subsequent discussions on optimization, including goal programming and fuzzy optimization models for production planning in uncertain environments. The inclusion of graph theory and fixed-point theory in metric spaces highlights the structural elegance of mathematics, while topics like ring theory, approximation theory, and hypersurfaces in metallic Riemannian manifolds delve into abstract and geometric frameworks.

Statistical methodologies are equally prominent, with a dedicated chapter on the fundamentals of statistics, complemented by advanced discussions on error analysis in interpolation methods and eigen value localization for quaternionic matrices. The book also addresses interdisciplinary applications, such as the study of magnetic field effects on rotating magneto-hydrodynamic (MHD) flows and hypergeometric transformations, which underscore the relevance of mathematics in physical and engineering contexts.

Our aim is to provide a cohesive yet diverse collection of topics that inspire further exploration and research. Each chapter is authored by experts in their respective fields, ensuring depth and clarity. Whether you are a mathematician, statistician, or practitioner in a related discipline, this book offers valuable insights into the theoretical underpinnings and practical applications of advanced mathematics and statistics.

We hope that *Advanced Studies in Mathematics and Statistics-I Volume-1* serves as a catalyst for intellectual curiosity and fosters a deeper appreciation for the beauty and utility of these disciplines. We invite readers to engage with the material, explore its applications, and contribute to the ongoing advancement of mathematical and statistical sciences.

Editor

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A FUZZY OPTIMIZATION MODEL FOR MULTI-ITEM PRODUCTION PLANNING PROBLEM IN AN UNCERTAIN ENVIRONMENT

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Abstract

In this chapter, we have formulated a multi-objective production planning model for a hardware firm. This firm produces different types of hardware locks and other items in its production run. The objectives of the firm are to minimize the production cost, minimize the inventory holding cost and maximize the net profit subject to these set of realistic constraints. In the past, the production planning problem of a similar type has been formulated with exact information where the input information is precisely known to the decision-maker. However, in most of the real-life situations, the input information is not precisely known. In such situations, fuzzy set theory plays a vital role in modelling of the problem where the input data have some vagueness. The proposed model of production planning also been formulated under fuzzy environment. The triangular fuzzy number has been used to present the vagueness in the input information. The equivalent crisp form of the fuzzy model has been obtained by two different types of defuzzification approaches namely ranking function and α -cut approach respectively. Henceforth, the formulated models under fuzzy environment have been solved by the fuzzy goal programming approach.

Keywords Production Planning Problem; Multi-objective Optimization; Fuzzy Goal Programming; Fuzzy Set Theory.

1. Introduction

Production planning is considered as the transformation of raw material into finished goods by using different applications with the proper procedure under optimal costs and optimal consumption of raw materials. The primary goal of any production planning is to successfully understand the market situations, which satisfy the customer's