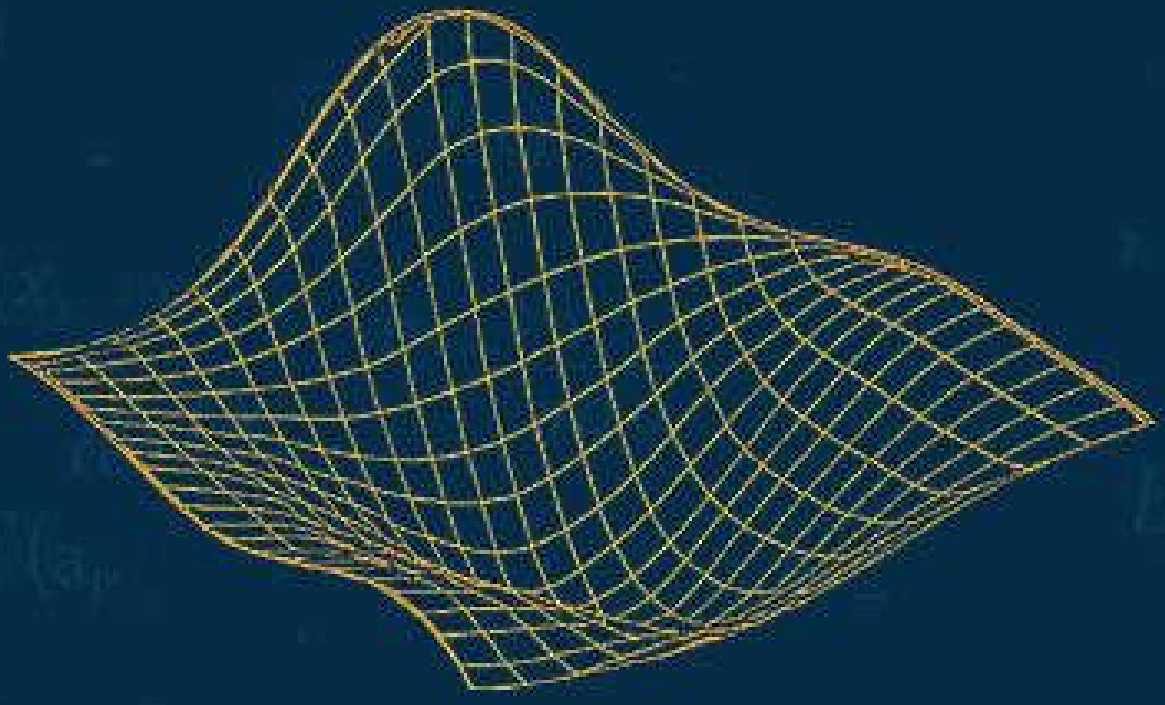
ADVANCED STUDIES IN MATHEMATICS AND STATISTICS

VOLUME-1



EDITOR
MOBIN AHMAD

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Mobin Ahmad

Department of Mathematics & Statistics
Integral University, Lucknow-226026
Email: mobin@iul.ac.in

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Author: Mobin Ahmad

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Preface

The field of mathematics and statistics continues to evolve, bridging theoretical advancements with practical applications across diverse disciplines. *Advanced Studies in Mathematics and Statistics-I Volume – 1* is a comprehensive volume that brings together cutting-edge research and foundational concepts in these fields, offering a rich resource for researchers, graduate students, and professionals seeking to deepen their understanding of advanced mathematical and statistical methodologies.

This book encompasses a broad spectrum of topics, carefully curated to reflect the dynamic interplay between pure and applied mathematics, as well as statistical theory. The chapters cover a range of subjects, from numerical methods for solving non-linear equations to the intricacies of graph theory, ring theory, and fuzzy optimization. Each chapter is designed to provide both theoretical rigor and practical insights, making the content accessible to those with a strong mathematical foundation while also serving as a reference for specialized research.

The book begins with an exploration of numerical solutions to non-linear equations, presenting classical and fast-convergent methods such as Newton's, Steffensen's, and Halley's approaches. These foundational techniques set the stage for subsequent discussions on optimization, including goal programming and fuzzy optimization models for production planning in uncertain environments. The inclusion of graph theory and fixed-point theory in metric spaces highlights the structural elegance of mathematics, while topics like ring theory, approximation theory, and hypersurfaces in metallic Riemannian manifolds delve into abstract and geometric frameworks.

Statistical methodologies are equally prominent, with a dedicated chapter on the fundamentals of statistics, complemented by advanced discussions on error analysis in interpolation methods and eigen value localization for quaternionic matrices. The book also addresses interdisciplinary applications, such as the study of magnetic field effects on rotating magneto-hydrodynamic (MHD) flows and hypergeometric transformations, which underscore the relevance of mathematics in physical and engineering contexts.

Our aim is to provide a cohesive yet diverse collection of topics that inspire further exploration and research. Each chapter is authored by experts in their respective fields, ensuring depth and clarity. Whether you are a mathematician, statistician, or practitioner in a related discipline, this book offers valuable insights into the theoretical underpinnings and practical applications of advanced mathematics and statistics.

We hope that *Advanced Studies in Mathematics and Statistics-I Volume-1* serves as a catalyst for intellectual curiosity and fosters a deeper appreciation for the beauty and utility of these disciplines. We invite readers to engage with the material, explore its applications, and contribute to the ongoing advancement of mathematical and statistical sciences.

Editor
Mobin Ahmad
Department of Mathematics & Statistics
Integral University, Lucknow
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FUNDAMENTAL RESULTS OF FIXED POINT THEORY IN METRIC SPACES

Uqba Rafat

Department of Mathematics & Statistics,

Faculty of Science, Integral University, Lucknow

226026, India

Email: **uqba@iul.ac.in**

1.1 Introduction

Fixed point theory continues to be one of the most powerful and popular tools of modern mathematics. We often come across with this concept in several mathematical domains such as Topology, Functional Analysis, Metric space, Operator Theory, Discrete Mathematics, Differential &Integral Equations including Inclusions, Theory of Fractals, Approximation Theory and similar others. The source of inspiration as well as the strength of fixed point theory is due to the numerous applications within and beyond the mathematics such as in: Biology, Computer Sciences, Economics, Engineering, Optimization Theory, Medical Sciences, Statistics and several others.

Given a non-empty set M, a point $r \in M$ is said to be fixed point of a self-mapping$T$ defined on M is r remains invariant under the mapping T, i.e., $Tr = r$. We denote the set the set of all fixed points of the mapping T by Fix(T). In other words, $Fix(T) \coloneqq \{ r \in M : Tr = r \}$. Several nonlinear problems can be reformulated to a problem of determining fixed points of mapping S defined on a suitable space M. We often come across with the equation of the form: $f(r) = 0$. To solve such equations, although there are several existing methods to find solutions (exact/approximate) but thefixed point method is most useful and effective tool to handle such problems.

For instance,

Let us consider a simple quadratic equation$f(r) = r^2 - 3r + 2 = 0$. It is easy to see that$r = 1$and $r = 2$are the roots of this equation. Now, if we rewrite this equation as: