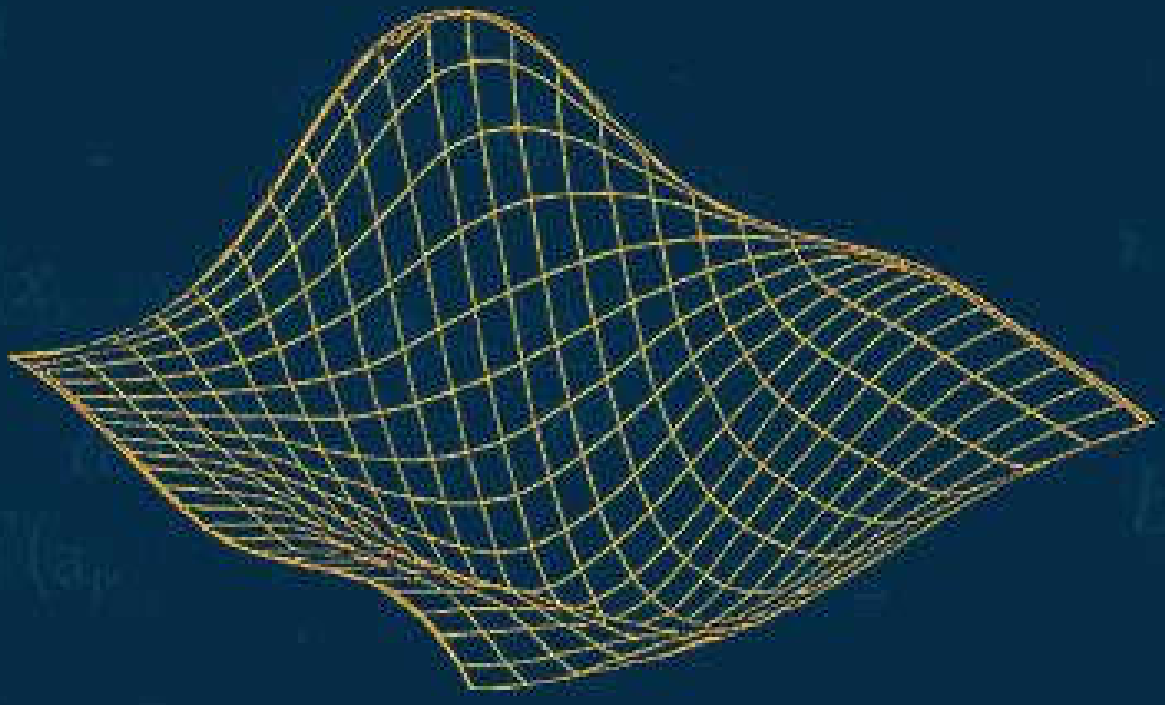
ADVANCED STUDIES IN MATHEMATICS AND STATISTICS

VOLUME-1



EDITOR
MOBIN AHMAD

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VOLUME-1

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Preface

The field of mathematics and statistics continues to evolve, bridging theoretical advancements with practical applications across diverse disciplines. *Advanced Studies in Mathematics and Statistics-I Volume – 1* is a comprehensive volume that brings together cutting-edge research and foundational concepts in these fields, offering a rich resource for researchers, graduate students, and professionals seeking to deepen their understanding of advanced mathematical and statistical methodologies.

This book encompasses a broad spectrum of topics, carefully curated to reflect the dynamic interplay between pure and applied mathematics, as well as statistical theory. The chapters cover a range of subjects, from numerical methods for solving non-linear equations to the intricacies of graph theory, ring theory, and fuzzy optimization. Each chapter is designed to provide both theoretical rigor and practical insights, making the content accessible to those with a strong mathematical foundation while also serving as a reference for specialized research.

The book begins with an exploration of numerical solutions to non-linear equations, presenting classical and fast-convergent methods such as Newton's, Steffensen's, and Halley's approaches. These foundational techniques set the stage for subsequent discussions on optimization, including goal programming and fuzzy optimization models for production planning in uncertain environments. The inclusion of graph theory and fixed-point theory in metric spaces highlights the structural elegance of mathematics, while topics like ring theory, approximation theory, and hypersurfaces in metallic Riemannian manifolds delve into abstract and geometric frameworks.

Statistical methodologies are equally prominent, with a dedicated chapter on the fundamentals of statistics, complemented by advanced discussions on error analysis in interpolation methods and eigen value localization for quaternionic matrices. The book also addresses interdisciplinary applications, such as the study of magnetic field effects on rotating magneto-hydrodynamic (MHD) flows and hypergeometric transformations, which underscore the relevance of mathematics in physical and engineering contexts.

Our aim is to provide a cohesive yet diverse collection of topics that inspire further exploration and research. Each chapter is authored by experts in their respective fields, ensuring depth and clarity. Whether you are a mathematician, statistician, or practitioner in a related discipline, this book offers valuable insights into the theoretical underpinnings and practical applications of advanced mathematics and statistics.

We hope that *Advanced Studies in Mathematics and Statistics-I Volume-1* serves as a catalyst for intellectual curiosity and fosters a deeper appreciation for the beauty and utility of these disciplines. We invite readers to engage with the material, explore its applications, and contribute to the ongoing advancement of mathematical and statistical sciences.

Editor
Mobin Ahmad
Department of Mathematics & Statistics
Integral University, Lucknow
Date: May 28, 2025

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PERTURBATION THEORY AND EIGENVALUE LOCALIZATION FOR QUATERNIONIC MATRICES

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This chapter presents a comprehensive exploration of perturbation theory and eigenvalue localization for matrices over the quaternion division algebra, addressing key theoretical and computational challenges arising from their non-commutative nature. Building on recent advances in quaternionic linear algebra, we extend classical perturbation results-such as the Bauer-Fike theorem and relative eigenvalue bounds-to the quaternionic setting, with a focus on both right and left eigenvalues, which exhibit fundamentally distinct behaviors. We derive new perturbation bounds for structured quaternionic matrices (e.g., Hermitian, unitary, and skew-Hermitian), leveraging their unique spectral properties. For non-diagonalizable matrices, we develop refined localization theorems using Schur and Jordan decompositions, providing tighter estimates for eigenvalue perturbations under additive, multiplicative, and nonlinear perturbations. Additionally, we investigate the perturbation of singular values and their role in quaternionic matrix conditioning. A significant emphasis is placed on applications, including the stability analysis of quaternionic linear systems and robust control design. We also extend eigenvalue localization techniques to the zeros of quaternionic polynomials, offering new insights for solving polynomial equations over quaternions. Computational aspects, such as scalable algorithms for estimating perturbation bounds, are discussed to bridge theory and practice. Finally, we highlight open problems and future directions, including extensions to octonionic matrices and connections to hypercomplex algebraic geometry. This chapter serves as a unified resource for researchers seeking to apply perturbation theory in quaternionic settings, with relevance to quantum mechanics, computer graphics, and aerospace engineering.