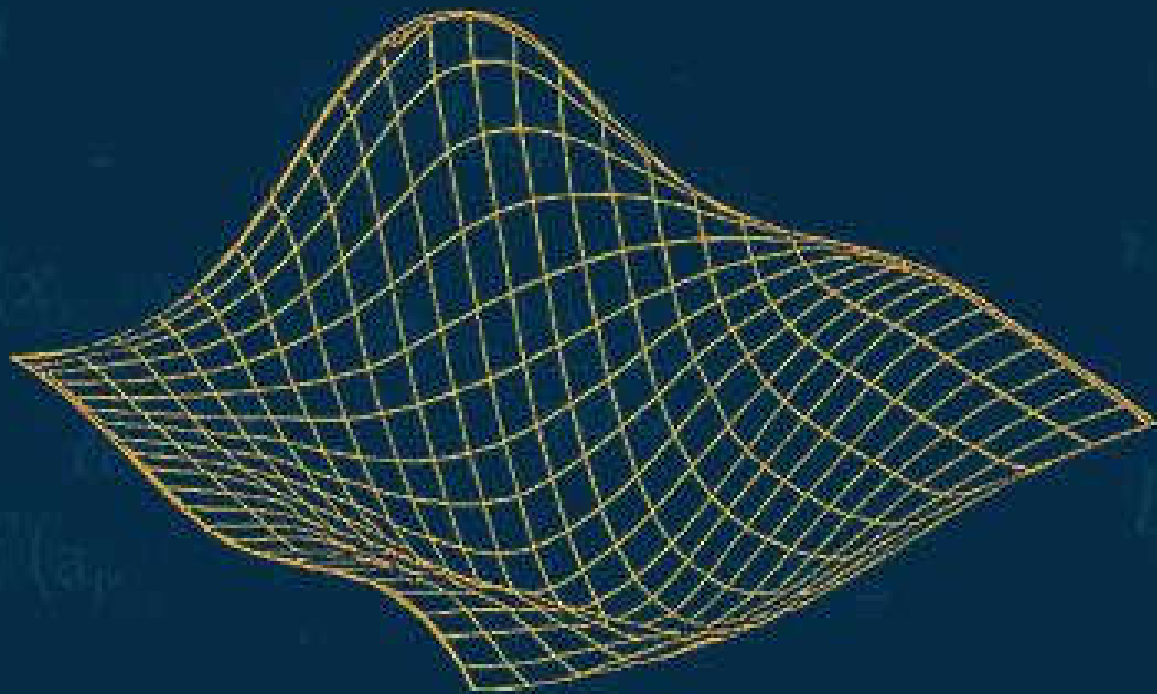


ADVANCED STUDIES IN MATHEMATICS AND STATISTICS

VOLUME-1



EDITOR **Activate Windows**

MOBIN AHMAD

ADVANCED STUDIES IN MATHEMATICS AND STATISTICS

**ADVANCED STUDIES IN
MATHEMATICS AND STATISTICS
VOLUME-1**

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Preface

The field of mathematics and statistics continues to evolve, bridging theoretical advancements with practical applications across diverse disciplines. *Advanced Studies in Mathematics and Statistics-I Volume – I* is a comprehensive volume that brings together cutting-edge research and foundational concepts in these fields, offering a rich resource for researchers, graduate students, and professionals seeking to deepen their understanding of advanced mathematical and statistical methodologies.

This book encompasses a broad spectrum of topics, carefully curated to reflect the dynamic interplay between pure and applied mathematics, as well as statistical theory. The chapters cover a range of subjects, from numerical methods for solving non-linear equations to the intricacies of graph theory, ring theory, and fuzzy optimization. Each chapter is designed to provide both theoretical rigor and practical insights, making the content accessible to those with a strong mathematical foundation while also serving as a reference for specialized research.

The book begins with an exploration of numerical solutions to non-linear equations, presenting classical and fast-convergent methods such as Newton's, Steffensen's, and Halley's approaches. These foundational techniques set the stage for subsequent discussions on optimization, including goal programming and fuzzy optimization models for production planning in uncertain environments. The inclusion of graph theory and fixed-point theory in metric spaces highlights the structural elegance of mathematics, while topics like ring theory, approximation theory, and hypersurfaces in metallic Riemannian manifolds delve into abstract and geometric frameworks.

Statistical methodologies are equally prominent, with a dedicated chapter on the fundamentals of statistics, complemented by advanced discussions on error analysis in interpolation methods and eigen value localization for quaternionic matrices. The book also addresses interdisciplinary applications, such as the study of magnetic field effects on rotating magneto-hydrodynamic (MHD) flows and hypergeometric transformations, which underscore the relevance of mathematics in physical and engineering contexts.

Our aim is to provide a cohesive yet diverse collection of topics that inspire further exploration and research. Each chapter is authored by experts in their respective fields, ensuring depth and clarity. Whether you are a mathematician, statistician, or practitioner in a related discipline, this book offers valuable insights into the theoretical underpinnings and practical applications of advanced mathematics and statistics.

We hope that *Advanced Studies in Mathematics and Statistics-I Volume-1* serves as a catalyst for intellectual curiosity and fosters a deeper appreciation for the beauty and utility of these disciplines. We invite readers to engage with the material, explore its applications, and contribute to the ongoing advancement of mathematical and statistical sciences.

Editor

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AN INTRODUCTION TO APPROXIMATION THEORY

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In this chapter, we have defined some basic ideas about the Approximation theory with their properties and applications. The approximation theory has a close relationship with other branches of mathematics (like Fourier analysis). Existence of such a relationship is explained by the fact that many important problems of the approximation theory are formulated.

The relationship between approximation theory and functional analysis is basically very close. In fact, all well-known methods of approximation of functions by means of algebraic or trigonometric polynomials (which are partial sum of interpolating polynomials, partial sums Of Fourier series, etc.) are linear operators.

The area of mathematics known as "approximation theory" examines how general functions can be approximated using simpler functions like polynomials, finite elements, or Fourier series. This Theory finds extensively applications in various areas like numerical analysis, engineering and computer science making it a corner stone for advancements in technology and scientific research. By understanding the fundamentals of approximation theory. Students can unlock new possibilities in computational methods and analytical problem solving.

The approximation theory has paved the way for whole new mathematical paths and aided advancement in other mathematical fields. Functional analysis and approximation theory have a very close link. The theory has been extensively studied in post quantum calculus recently with the introduction of the (p, q) analogues of linear positive operator, the determination of their approximation properties and the presentation of advantages based on classical theories.