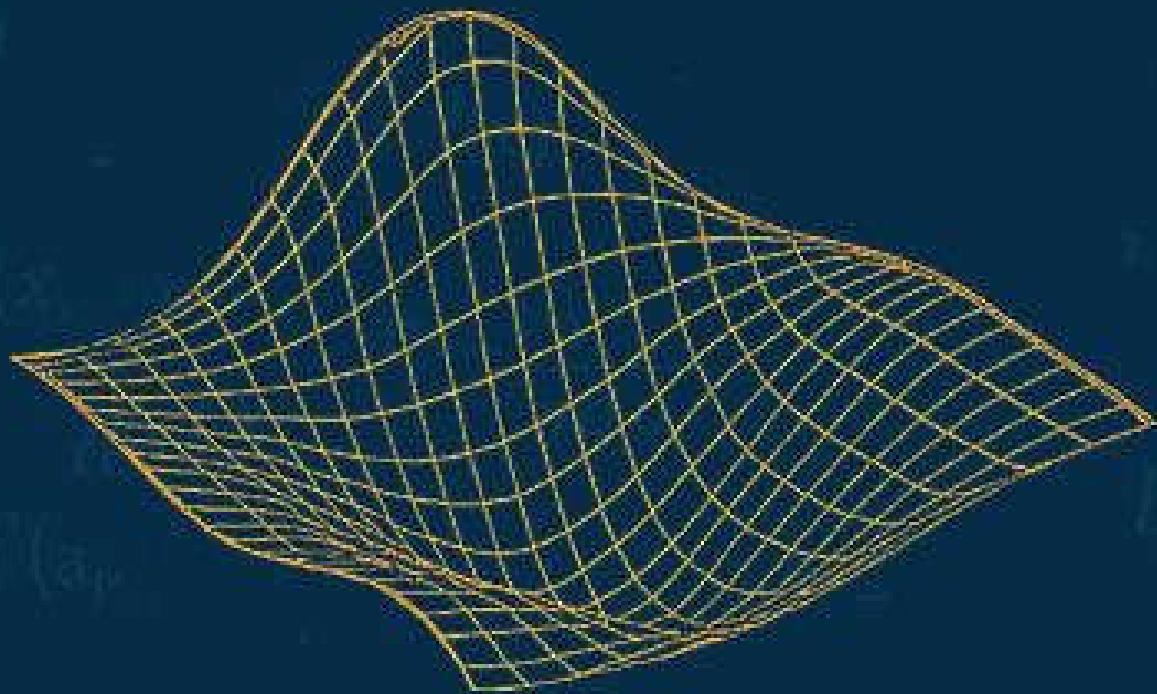


# ADVANCED STUDIES

## IN MATHEMATICS AND STATISTICS

VOLUME-1



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**MOBIN AHMAD**

ADVANCED STUDIES IN MATHEMATICS AND STATISTICS

**ADVANCED STUDIES IN  
MATHEMATICS AND STATISTICS  
VOLUME-1**

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# Preface

The field of mathematics and statistics continues to evolve, bridging theoretical advancements with practical applications across diverse disciplines. *Advanced Studies in Mathematics and Statistics-I Volume – I* is a comprehensive volume that brings together cutting-edge research and foundational concepts in these fields, offering a rich resource for researchers, graduate students, and professionals seeking to deepen their understanding of advanced mathematical and statistical methodologies.

This book encompasses a broad spectrum of topics, carefully curated to reflect the dynamic interplay between pure and applied mathematics, as well as statistical theory. The chapters cover a range of subjects, from numerical methods for solving non-linear equations to the intricacies of graph theory, ring theory, and fuzzy optimization. Each chapter is designed to provide both theoretical rigor and practical insights, making the content accessible to those with a strong mathematical foundation while also serving as a reference for specialized research.

The book begins with an exploration of numerical solutions to non-linear equations, presenting classical and fast-convergent methods such as Newton's, Steffensen's, and Halley's approaches. These foundational techniques set the stage for subsequent discussions on optimization, including goal programming and fuzzy optimization models for production planning in uncertain environments. The inclusion of graph theory and fixed-point theory in metric spaces highlights the structural elegance of mathematics, while topics like ring theory, approximation theory, and hypersurfaces in metallic Riemannian manifolds delve into abstract and geometric frameworks.

Statistical methodologies are equally prominent, with a dedicated chapter on the fundamentals of statistics, complemented by advanced discussions on error analysis in interpolation methods and eigen value localization for quaternionic matrices. The book also addresses interdisciplinary applications, such as the study of magnetic field effects on rotating magneto-hydrodynamic (MHD) flows and hypergeometric transformations, which underscore the relevance of mathematics in physical and engineering contexts.

Our aim is to provide a cohesive yet diverse collection of topics that inspire further exploration and research. Each chapter is authored by experts in their respective fields, ensuring depth and clarity. Whether you are a mathematician, statistician, or practitioner in a related discipline, this book offers valuable insights into the theoretical underpinnings and practical applications of advanced mathematics and statistics.

We hope that *Advanced Studies in Mathematics and Statistics-I Volume-I* serves as a catalyst for intellectual curiosity and fosters a deeper appreciation for the beauty and utility of these disciplines. We invite readers to engage with the material, explore its applications, and contribute to the ongoing advancement of mathematical and statistical sciences.

## **Editor**

Mobin Ahmad

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Date: May 28, 2025

# Contents

<b>S. No</b>	<b>Title</b>	<b>Page No</b>
1.	Numerical Solutions of Non-Linear Equations by Newton's, Steffensen's & Halley's Method	2-22
2.	Non-linear Equations and Their Numerical Solution by Fast Convergent Methods	24-35
3.	Goal Programming	37-50
4.	Graph Theory	52-72
5.	Comprehensive Overview of Ring Theory and Polynomial Structures	74-85
6.	An Introduction to Approximation Theory	87-99
7.	Hypersurfaces Immersed in Metallic Riemannian Manifolds	101-113
8.	Fuzzy Ideals in Ordered Semigroups	115-132
9.	Perturbation Theory and Eigenvalue Localization for Quaternionic Matrices	134-159
10.	Fundamentals of Statistics	161-180
11.	Fundamental Results of Fixed-Point Theory in Metric Spaces	182-189
12.	Hypergeometric Transformations and Their Applications	191-200
13.	Magnetic Field Effects on Rotating Mhd Flow Over an Impulsively Started Isothermal Plane	202-212
14.	A Fuzzy Optimization Model for Multi-Item Production Planning Problem in an Uncertain Environment	214-237
15.	Exploring The Accuracy of Interpolation Methods Through Error Analysis	239-249

# COMPREHENSIVE OVERVIEW OF RING THEORY AND POLYNOMIAL STRUCTURES

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This chapter offers a unified and paraphrased examination of essential concepts in ring theory, with a particular focus on polynomial rings. It introduces core algebraic definitions, explores fundamental theorems along with their rigorous proofs, and incorporates illustrative examples to clarify theoretical insights. The purpose of this chapter is to introduce and define the basic principles and preliminary concepts that are foundational for further study in ring theory. These introductory materials serve as a basis for the subject matter. The content is largely based on standard algebraic references.

## 1. Basic Definitions

The present section deals with basic notions in the theory of rings which shall be required for the development of the subject in subsequent chapters. Examples are also included in this section to make the matter presented in the section self-explanatory and to give a clear sketch of the various notions.

### Definition 1.1 Ring

A ring  $R$  is a non-empty set equipped with two binary operations: addition (+) and multiplication ( $\cdot$ ). For all elements  $a$ ,  $b$ , and  $c$  in  $R$ , the following properties hold:

- Addition (+) forms an abelian group:
  1. Associativity:  $(a + b) + c = a + (b + c)$
  2. Identity: There exists  $0 \in R$  such that  $a + 0 = a$
  3. Inverses: For each  $a \in R$ , there exists  $-a$  such that  $a + (-a) = 0$
  4. Commutativity:  $a + b = b + a$
- Multiplication ( $\cdot$ ) satisfies:
- ..
  1. Associativity:  $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
  2. Distributivity:  $a \cdot (b + c) = a \cdot b + a \cdot c$  and  $(a + b) \cdot c = a \cdot c + b \cdot c$